Lattice QCD and the Jefferson Lab Program

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Abstract. Lattice gauge theory provides our only means of performing \textit{ab initio} calculations in the nonperturbative regime. It has thus become an increasingly important component of the Jefferson Lab physics program. In this paper, we describe the contributions of lattice QCD to our understanding of hadronic and nuclear physics, focusing on the structure of hadrons, the calculation of the spectrum and properties of resonances, and finally on deriving an understanding of the QCD origin of nuclear forces.

1. Introduction

Lattice QCD currently provides our only means of solving QCD in the low-energy regime, and thus has a natural place in the program of Jefferson Lab. Nathan Isgur recognised the potential of lattice QCD to greatly enhance our understanding of hadronic and nuclear physics when he established, in collaboration with John Negele of MIT, a lattice initiative at the laboratory. That initiative spans the three major components of a successful lattice effort. Firstly, the development of the computational and software infrastructure conducted by both the Theory Center and the High-Performance Computing Group under the auspices of the Department of Energy’s SciDAC (\textit{Scientific Discovery through Advanced Computing}) Initiative. Secondly, the development and operation of hardware, based on commodity clusters and latterly heterogeneous architectures, optimized for lattice QCD, and now serving as a national facility. Finally, a rigorous research effort dedicated to addressing the key scientific questions posed by the Jefferson Lab program.

Lattice gauge calculations solve QCD on a four-dimensional lattice, or grid, of points in Euclidean space. The quarks reside on the points of the grid, whilst the gluons are associated with the links joining those points. Lattice calculations proceed through a Monte Carlo method, in which ensembles of gauge configurations are generated with a probability distribution prescribed by the Euclidean QCD action. At the start of the lattice program at Jefferson Lab, most lattice calculations were performed in the so-called quenched approximation to QCD, in which quarks would propagate through a gluonic sea, but themselves did not affect the distribution of the gluons. Such an approximation reduced the computational cost by several orders of magnitude and had some theoretical justification, notably from large-$N_c$ QCD. The claim to be \textit{solving}, rather than modelling, QCD has only been truly justified with the elimination of the quenched approximation.

The remainder of this paper is laid out as follows. We conclude the introduction with an outline of some of the algorithmic and computational advances that have taken place at the laboratory. We then describe the lattice studies of hadron structure, including those of
generalized parton distributions (GPDs) and of the pion form factor. The next section focuses on the resonance spectrum of QCD, and the relation of the program both to the current search for baryon resonances, and the future quest to photoproduce exotics at GlueX. We then describe how lattice QCD can be used to delve into the origin of the nuclear force. Finally, we conclude with the prospects for lattice QCD in the 12 GeV era.

2. Computation challenges
Lattice QCD calculations have historically been at the leading edge of exploiting the most powerful supercomputing resources available. In part, this is due to the natural enthusiasm of the proponents of the field, but is also conditioned by the highly regular nature of the problem. The lattice can be decomposed across a grid of processing nodes, with each node responsible for a given sub-lattice and executing exactly the same code, and with communication of the surfaces of the sub-lattices to the neighboring nodes; the problem is inherently data-parallel. The cost of most lattice calculations is dominated by the solution of a large, sparse system of equations, corresponding to the discretized Dirac equation, and therefore an important task for lattice practitioners is the development of efficient solvers, both for the generation of the gauge configurations, and for the subsequent measurement of physics observables. An important advance for the latter has been the development of eigenvalue deflation methods, enabling physics measurements to be made with greatly increased efficiency [1].

Our ability to exploit the latest and most powerful architectures benefits strongly by the availability of portable software that can be optimized for the target platform. The development of such software has been the aim of the National Computational Infrastructure for Lattice Gauge Theory funded under the SciDAC initiative. A particularly notable achievement has been the development, principally at Jefferson Lab, of the chroma software suite [2], which has been adopted by many lattice gauge groups both within the US and worldwide.

The development of high-performance, cost-optimized facilities for lattice QCD calculations has been an important activity of the High-Performance Computing Group. The first Intel-based cluster was installed in 2002, and Jefferson Lab now provides a national facility for lattice QCD, funded by the Department of Energy (DOE); the cluster installed in 2007, comprising nearly 400 nodes of quad-core AMD Opteron CPUs connected via Infiniband, is shown in Figure 1.

Recently, General Purpose Graphical Processing Units (GPGPUs) have become available that provide significant floating point performance at a relatively low cost. As part of the 2009 and 2010 ARRA project in lattice QCD, Jefferson Lab deployed a cluster with NVIDIA GPUs. A single GPU achieved 178 Gflops in single precision for one of the key numerical kernels, the inverter, used in lattice QCD. By comparison, a cluster node with dual quad-core CPUs could achieve only 18 Gflops. Thus GPUs admitted a ten-fold increase in computational performance for the inverter compared with that obtained on the host node at a relatively small increment in cost.

In order to perform calculations on the spatial volumes needed to confront experiment, it is necessary to spread the calculation across more than a single CPU. Researchers from Boston University, Harvard University and Jefferson Lab developed a parallel lattice QCD solver running on multiple GPUs across multiple cluster nodes. With this development, GPUs are now central to the lattice effort at the laboratory, enabling lattice calculations to address a range of questions previously inaccessible.

3. Hadron structure
The ability for lattice QCD to make increasingly precise, ab initio calculations of hadron structure has been driven by advances in three areas: algorithmic advances, enabling both chiral symmetry to be more faithfully represented on the lattice and gauge configurations to
be generated more efficiently, computational advances represented by the availability of special-purpose and leadership-class computers and by commodity clusters, and theoretical advances enabling extrapolations to be performed from the quark masses employed in the computation to the physical up- and down-quark masses. Members of the Theory Center, in collaboration with the Lattice Hadron Physics Collaboration (LHPC), have been at the forefront of applying these advances to aid our understanding of hadron structure and to the benefit of the Jefferson Lab program. The layout of the rest of this section is as follows. We begin by describing how hadron-structure calculations proceed in lattice QCD. We will then review benchmark calculations, such as that of the nucleon’s electromagnetic form factors. Subsection 3.3 will describe calculations of moments of generalized parton distributions and the impact such calculations are having on our understanding of hadron structure. We will conclude with prospects for future calculations.

3.1. Anatomy of a nucleon-structure calculation

Hadron structure is expressed through quantities such as the electromagnetic form factors, describing the distribution of charge and currents within a hadron, and the polarized and unpolarized parton structure functions, describing how the longitudinal momentum fraction and spin is apportioned amongst the constituents. Generalized parton distributions [3, 4, 5], encompass both of these concepts, and allow a three-dimensional picture of the nucleon to be constructed.

GPDs are expressed as the matrix elements of light-cone correlation functions $O_\Gamma(x)$:

$$O_\Gamma(x) = \int d\lambda e^{i\lambda x} \bar{q} \left( \frac{-\lambda n}{2} \right) \Gamma \mathcal{P} e^{-ig f_{N/2}^n a_m A(a_m) q \left( \frac{\lambda}{2} \right)}$$

where $n$ is a light-cone vector, the parallel-transporter is necessary to ensure gauge invariance,
and the flavor indices on the quarks are suppressed. The familiar quark polarized and unpolarized parton distributions are then forward matrix elements of this operator, with \( \Gamma = \gamma_{\mu} \) or \( \gamma_{\mu}\gamma_{5} \) for the unpolarized and polarized distribution functions, respectively; the GPDs correspond to matrix elements in the off-forward direction, with different momenta for the incoming and outgoing partons.

The use of a Euclidean lattice precludes the measurement of these matrix elements directly. Instead we appeal to the operator product expansion to expand the operator \( \mathcal{O}(x) \) about the light cone, yielding a set of local operators which can be measured on a Euclidean lattice, and furthermore analytically continued to Minkowski space. In particular, the moments, with respect to Bjorken-\( x \), of the quark distributions are obtained in terms of the forward matrix elements

\[
\mathcal{O}_{\Gamma}^{\{\mu_{1} \ldots \mu_{n}\}} = \bar{q}(\gamma_{5})\gamma^{\{\mu_{1}}\, iD_{\mu_{2}} \ldots D_{\mu_{n}}\} q.
\]  

(2)

Specifically, for the unpolarized distribution for a nucleon carrying momentum \( \vec{p} \), we have

\[
\langle \vec{p}|\mathcal{O}_{q}^{\mu_{1} \ldots \mu_{n}}|\vec{p}\rangle \longrightarrow \int_{0}^{1} dx \, x^{n-1} q(x),
\]  

(3)

where the Lorentz structure on the right-hand-side is suppressed.

The spectrum is obtained in lattice QCD through the measurement of so-called two-point functions representing the Euclidean correlators of two operators that interpolate between a state and the vacuum. Nucleon structure is investigated through the measurement of three-point functions, illustrated in Figure 2, representing the Euclidean correlation functions of three operators:

\[
C_{3pt}(t_{snk}, \tau; \vec{p}, \vec{q}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p} \cdot \vec{x} - i\vec{q} \cdot \vec{y}} \langle 0|J(\vec{x}, t_{snk})\mathcal{O}(\vec{y}, \tau)J(0)|0\rangle,
\]  

(4)

where \( J \) is an operator that interpolates between the state and the vacuum. Note that the disconnected contribution in the right-hand panel of Figure 2 is far more computationally demanding that the connected piece; for most of the following, we will emphasise the calculation of flavor non-singlet, or isovector, quantities, for which the disconnected piece vanishes. We insert complete sets of states between the operators in Eq. (4); the time-slice sum projects onto states of definite momentum, and we obtain:

\[
C_{3pt}(t_{snk}, \tau; \vec{p}, \vec{q}) = \sum_{n=1, \ldots} e^{-E_{n}(\vec{p})}(t_{snk} - \tau)e^{-E_{n}(\vec{p} + \vec{q})\tau} \times \langle n, \vec{p}|\mathcal{O}(0)|n, \vec{p} + \vec{q}\rangle\langle n, \vec{p} + \vec{q}|J(0)|0\rangle.
\]  

(5)

For sufficiently large \( \tau \) and \( t_{snk} - \tau \), \( C_{3pt} \) is dominated by the lowest-lying state, and we can obtain \( \langle n = 1, \vec{p}|\mathcal{O}|n = 1, \vec{p}\rangle \) after elimination of the vacuum-to-state matrix elements from the corresponding two-point functions.

Computations of hadron structure are being performed by several groups, using a variety of fermion and gauge-field discretizations.

The common theme is calculations at decreasingly small values of the pion mass, enabling lattice calculations, together with chiral effective theory, to provide insight into physics at the physical quark masses; recent reviews are contained in Ref. [6, 7]. In the remainder of this section, we will in large part quote results from the LHP Collaboration. We employ a hybrid approach, using lattices generated by the MILC Collaboration [8] with \( 2 + 1 \) flavors of Asqtad quarks, corresponding to degenerate \( u/d \) and strange, but with domain-wall fermions (DWF) for the valence quarks, computed on HYP-smereared lattices [9]. The generation of lattices using
3.2. Electromagnetic form factors

These describe the distribution of charge and current within the nucleon, and remain the subject of intense experimental and theoretical interest. They are the most straightforward quantity to measure in a lattice calculation, corresponding to the matrix element of the electromagnetic current $V_\mu = \bar{q} \gamma_\mu q$. Figure 3 shows the form factor $F_1(Q^2)$ determined by the LHP Collaboration [11], together with a parametrization of the isovector experimental data [12]. The data show the approach to the experimental parametrization at decreasing values of the pion mass used in the lattice calculation.

The slope of the form factor at $Q^2 = 0$ is related to the isovector charge radius $(\langle r^2 \rangle)^{1/2}$. The right-hand panel in Figure 3 shows the charge radius obtained from the calculation above. A naive linear fit in $m_\pi^2$ would clearly be far below the experimental points, illustrating the need to

Asqtad fermions is very computationally efficient, whilst the DWF valence quarks have the desired chiral-symmetry properties, simplifying operator mixing and hence the matching of our results to the continuum. Whilst this approach violates unitarity at finite lattice spacing, we expect this approach to yield the correct theory in the continuum limit [10].
correctly describe the non-analytic behaviour in the approach to the physical light-quark masses. The curve shows the chiral extrapolation of the charge radius, using the finite-range regulator [13].

An important corollary to this work has been the investigation of the electromagnetic properties of baryons containing the strange valence quark [14]. The computation of the contribution of the strange quarks, and more generally sea quarks, to hadron structure is very challenging. However, the combination of lattice QCD, the constraints of charge symmetry and chiral-extrapolation techniques has enabled the strange-quark contributions to the electric and magnetic form factors to be determined with high precision [15, 16], providing an important theoretical complement to the experimental measurements.

3.3. Generalized parton distributions

The electromagnetic form factors described above are particular cases of generalized parton distributions. There are two GPDs corresponding to the choice \( \Gamma = \gamma \cdot n \) in Eq. (1), \( H(x, \xi, t) \) and \( E(x, \xi, t) \), and a further two corresponding to the \( \Gamma = \gamma \cdot n\gamma_5 \), \( \tilde{H}(x, \xi, t) \) and \( \tilde{E}(x, \xi, t) \). The invariants are:

\[
\begin{align*}
t &= -\Delta^2 = -(P - P')^2, \\
\xi &= -n \cdot \Delta/2,
\end{align*}
\]  

where \( n_\mu \) is a light-cone vector, and \( P, P' \) are the four-momenta of the incoming and outgoing states. Once again, we appeal to the operator product expansion to obtain the matrix elements of local operators, and both the form factors described above and the familiar parton distributions emerge as special cases:

\[
\begin{align*}
H(x, 0, 0) &= q(x), \\
\tilde{H}(x, 0, 0) &= \Delta q(x),
\end{align*}
\]

and

\[
\int_{-1}^{1} dx H(x, \xi, t) = F_1(t).
\]

Note here that the \( x \) runs from \(-1\) to \(1\), corresponding to antiquark and quark momentum fraction.

Before proceeding to discuss the new insights enabled by the study of GPDs, we begin by discussing the polarized and unpolarized distributions as a means of benchmarking our calculation. A fundamental measure of QCD is the nucleon’s axial-vector charge \( g_A \), corresponding to the choice \( \Gamma = \gamma_\mu \gamma_5 \); it has additional importance in its rôle as a fundamental low-energy constant of the theory that appears in the chiral expansion of other quantities. Figure 4 shows a calculation of the axial-vector charge by the LHP Collaboration, together with the chiral extrapolation to the physical quark masses [17]. The consistency between the extrapolated lattice data and the experimental value is very encouraging, though the chiral behaviour for this quantity is very mild.

The non-analytic behaviour with decreasing quark mass is more strongly exhibited in the unpolarized distributions, and in particular in the calculation of the momentum fraction carried by the valence quarks in the nucleon. An analysis of the forward matrix elements for both the unpolarized and polarized distributions, as well as for the transversity, has been performed using a renormalized chiral expansion [21]; data for the non-singlet momentum fraction, and a summary of the results for a range of benchmark quantities, is shown in Figure 5.
Figure 4. The squares show LHPC/MILC data for the axial-vector charge $g_A$; the two data points at the lowest pion mass correspond to to different volumes. The remaining lattice points are taken from QCDSF/UKQCD [18], RBCK [19] and SESAM [20].

Figure 5. The left-hand figure shows the momentum fraction carried by the valence quarks in a nucleon, together with the chiral extrapolation to the physical quark masses. The right-hand figure is a summary which shows the lattice values at the physical pion masses for a range of benchmark quantities, normalized to the experimental value where known (left-hand points) [21]; the errors on the corresponding experimental measurements are shown to the right of the experimental points.

3.4. New insights: origin of nucleon spin

The excitement and significance of GPDs arises from the new insights they can provide into hadron structure, spurring new experimental initiatives such as the 12 GeV upgrade at Jefferson Lab. There is thus a rigorous effort both by the LHP Collaboration in a hybrid DWF/Asqtad approach [22, 21], and by the QCDSF/UKQCD Collaboration using two flavors of dynamical, improved Wilson quarks [23], to measure moments of GPDs.

A salient example of the new insights facilitated through the study of GPDs is how the spin of the nucleon is distributed amongst its constituents, and in particular the rôle of orbital angular momentum of the quarks. The spin carried by the quarks in a nucleon has long been the pursuit of lattice calculations, but a means to calculate the orbital angular momentum of the quarks eluded theorists. However, it was realized that the total angular momentum carried by the
quarks within a nucleon could be related to moments of GPDs through \[ J_q = \frac{1}{2} \int dx \, x (H(x, \xi, 0) + E(x, \xi, 0)) \].

A pioneering effort to measure the quark orbital angular momentum was performed in the quenched approximation to QCD [25]; Figure 6 shows a more recent calculation by the LHP Collaboration. Note that the calculation includes only the connected pieces. None-the-less, it suggests that the total orbital angular momentum carried by the quarks within a nucleon is small, but that the orbital angular momentum carried by the individual quark flavors may be substantial.

### 3.5. Pion form factor
The measurement of the form factor of the pion is an important part of the Jefferson Lab program both at 6 GeV and after the future 12 GeV upgrade. The pion is the simplest, and lightest, hadron and provides an important theatre in which to test many of our ideas about hadron structure, notably the range of validity of vector-meson dominance with increasing \( Q^2 \), and the approach to asymptotic freedom in form factors at high \( Q^2 \). The experimental study of the pion form factor is obscured by the lack of a free pion target, except at very low momenta, requiring an extrapolation to the pion pole. In contrast, the study of pion structure in lattice QCD is subject to no such ambiguities.

The pion form factor in the hybrid calculation using DWF for the valence quarks and the asqtad formulation for the sea quark is shown in Figure 7 [28]. We see that the lattice calculation is able to explore a momentum region beyond that explored experimentally, and the lattice data are well described by vector-meson dominance over the region of the calculation. Also shown is the expectation for the asymptotic behavior of the form factor, it is clear that the lattice calculation is far from the perturbative regime at the current momentum transfers.

### 4. Spectroscopy
In order to really understand QCD and hence test whether it is the complete theory of the strong interaction, we must know the spectrum of mesons and baryons that it implies and test those spectra against high quality data. The complete combined analysis of available experimental data on the photoproduction of nucleon resonances was the 2009 milestone in Hadronic Physics (HP), and the measurement of the electromagnetic properties of the low-lying baryons is an HP 2012 milestone. The observed spectrum of QCD provides little direct evidence of the presence
of the gluons. However, QCD admits the possibility of exotic mesonic states of matter in which the gluonic degrees of freedom are explicitly exhibited. The search for such states will be an important component of the Jefferson Lab 12 GeV program.

Given the intense experimental efforts in hadron spectroscopy, the need to predict and understand the hadron spectrum from first principles calculations in QCD is clear. Hence, an important goal of the lattice effort since its inception has been the study of the resonance spectrum of QCD, and more recently the calculation of the electromagnetic properties of low-lying resonances.

A comprehensive picture of resonances requires that we go beyond a knowledge of the ground state mass in each channel, and obtain the masses of the lowest few states of a given quantum number. This we can accomplish through the use of the variational method [30, 31]. Rather than measuring a single correlator \(C(t)\), we determine a matrix of correlators

\[
C_{ij}(t) = \sum_{\vec{x}} \langle O_i(\vec{x}, t) O_j^\dagger(\vec{0}, 0) \rangle,
\]

where \(\{O_i; i = 1, \ldots, N\}\) is a basis of interpolating operators with given quantum numbers. We then solve the generalized eigenvalue equation

\[
C(t) u = \lambda(t, t_0) C(t_0) u
\]

to obtain a set of real (ordered) eigenvalues \(\lambda_n(t, t_0)\), where \(\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{N-1}\). At large Euclidean times, these eigenvalues then delineate between the different masses

\[
\lambda_n(t, t_0) \longrightarrow e^{-M_n(t-t_0)} + \mathcal{O}(e^{-\Delta M_n(t-t_0)}),
\]

where \(\Delta M_n = \min\{| M_n - M_i |; i \neq n\}\). The eigenvectors \(u\) are orthogonal with metric \(C(t_0)\), and a knowledge of the eigenvectors can yield information about the partonic structure of the states.

The (hyper-) cubic lattice does not possess the full rotational symmetry of the continuum. Thus in a lattice calculation, states at rest are classified not according to the spin \((J, J_z)\), but rather according to the irreducible representations (irreps) of a cube; for states of higher spin, the different continuum degrees of freedom are distributed across several lattice irreps.

**Figure 7.** The open symbols show the pion form factor computed in a hybrid DWF/Asqtad calculation at two different values of light quark mass; the shaded bands are VMD fits to the calculation. The closed symbols show the experimental data, whilst the dashed line is the perturbative QCD expectation [29].
Thus a crucial ingredient in our spectroscopy program has been the use of a large basis of interpolating operators, decomposed into their lattice irreps, enabling the many spins to be reliably determined. Our ability to calculate correlation functions efficiently for such a large basis of operators has been greatly advanced through the development of a new method, “distillation” [32], for the construction of the interpolating operators.

The final ingredient in the spectroscopy effort has been the generation of so-called anisotropic lattices, with finer temporal than spatial discretization, enabling the fall-off of the principle correlators introduced above to be discerned at short temporal separations.

4.1. Anisotropic clover lattice generation
Precise calculations that can truly confront the experimental program require that we generate lattice with the correct number of light-quark flavors. The use of anisotropic lattices has proved essential in the reliable determination of the energy levels necessary to extract the resonance spectrum. Thus a crucial activity has been the generation of lattices with two flavors of fully dynamical light and a dynamical strange quark, using the clover fermion action, designed both for studies of the spectrum, and for the calculation of the scattering lengths important for understanding the nucleon-nucleon interaction. An important milestone was the tuning of the parameters of the action, beginning with the three-flavor theory [33].

A major challenge in calculations in lattice QCD is a procedure for specifying the values of the quark masses, and in particular those of the light $u/d$ and $s$ quarks, in a way that enables lattice calculations to be extrapolated to the physical values of these masses. This was accomplished through the introduction of a novel pair of dimensionless coordinates $l_\Omega$ and $s_\Omega$ that are primarily sensitive to the light and strange quark masses, respectively [34]:

$$l_\Omega = \frac{9m_\pi^2}{4m_\Omega^2},$$

$$s_\Omega = \frac{9(2m_K^2 - m_\pi^2)}{4m_\Omega^2}. \quad (11)$$

The use of these coordinates is illustrated in Figure 8, showing the approach at fixed strange-quark mass to the physical values of the light ($u/d$) quark masses.

Whilst the anisotropic lattices are designed to enable the investigation of the resonance spectrum of QCD, the low-lying spectrum, corresponding to the lightest states for each
quantum number, provides an important benchmark for future calculations. The left-hand panel of Figure 9 shows a summary of the low-lying light-hadron masses compared with their experimental values; the right-hand panel shows the relative errors on the results compared to experiment. It is this lattice-generation program that is at the core of our efforts to understand QCD spectroscopy, and has been a key element of the national USQCD program under the DOE INCITE awards since 2007.

4.2. Meson resonances
The new Hall D of the Jefferson Lab 12 GeV upgrade centers on the study of meson states produced in photoproduction reactions in the GlueX. Photoproduction has been proposed, within QCD-motivated models, as a favorable method for the production of exotic hybrid mesons, those mesons having $J^{PC}$ outside the set allowed to a fermion-antifermion pair. The hybrid hypothesis is that an excited gluonic field in addition to a quark-antiquark pair can give rise to these quantum numbers.

Over the past few years the Jefferson Lab lattice group has developed a methodology for the extraction of the spectrum and photocouplings including those involving excited states and exotics. The initial calculations were performed using charm mass quarks [35]. The charmonium system is particularly attractive, in that it is both computationally far less demanding than systems composed of the light ($u,d,s$) quarks, and because there is a wealth of high-precision experimental data. These calculations were performed in the quenched approximation to QCD, using anisotropic lattices; here the use of an anisotropic lattice, with temporal lattice spacing $a_t < a_s$, performs a further rôle in controlling the discretization uncertainties, $a_t m_c \ll 1$.

A formidable barrier to the interpretation of lattice calculations has been the reduced cubic symmetry imposed by the lattice which makes the assignment of the continuum spins to the energy eigenstates problematic. We discovered that, by judiciously constructing operators so as to have a known continuum behavior, the spins of the excited states could be determined, and the barrier imposed by the reduced cubic symmetry of the lattice overcome, and this discovery has been core both to investigation of the meson spectrum in both the light- and charm-quark sectors.

The availability of the large ensembles of $N_f = 2 + 1$ anisotropic clover lattices, together with the efficient operator-construction admitted by “distillation”, has enabled us to compute the isovector meson spectrum for states composed of the light and strange quarks [36, 37], at pion masses down to around 400 MeV. Masses of highly excited states were calculated, with the
accessible to the GlueX experiment. Results are important: there should be mesons of exotic quantum numbers in a mass region.

Figure 11 summarizes the results on exotic states and compares those results with previous lattice QCD results from Refs. [38, 39, 40, 41, 42, 43]. The experimental implications of these results are important: there should be mesons of exotic quantum numbers in a mass region accessible to the GlueX experiment.

Lattice QCD can go beyond the calculation of the spectrum to compute the transition
Figure 12. The lattice calculation of the radiative transition form factor; the band is a quark-model fit to the lattice data, yielding the value for the photocoupling at $Q^2 = 0$. Also shown are the experimental extractions of the photocouplings.

Figure 13. First and second excited vector state electric dipole transition form factors to the ground state scalar in charmonium. Experimental results for real photons from PDG and CLEO-c.

Form factors between states, and the photocouplings, that can guide experiment as to expected photoproduction rates. The calculation of the transition form factors follows the same procedure as that used to study hadron structure, and once again the charmonium system provides an important theatre both to develop the tools and to compare with experiment. Figure 12 shows the radiative transition form factor $J/\psi \to \eta_c \gamma$, obtained using domain-wall fermions on an anisotropic lattice [44]. Also shown is value for the photocoupling obtained from the experimental measurement of the partial width; the more recent 2008 analysis by CLEO-c was inspired by this lattice calculation. The optimum excited state operators determined from the study of the resonance spectrum can be used to extract photocouplings involving excited and exotic mesons [45], some examples of which are presented in Figure 13. Most importantly, this paper showed, for the first time, that radiative transitions to states with exotic quantum numbers were calculable, and furthermore extracted a large partial width for the exotic decay $\Gamma(\eta_{c1} \to J/\psi \gamma)$; should these results be continued to the light-quark sector, they would suggest copious photoproduction rates for exotic mesons.

4.3. $N^*$ resonances in lattice QCD

Baryons, containing three quarks, are emblematic of the non-Abelian nature of QCD, and of the three colors of the theory. An important goal in exploring baryons is attempting to discern the effective degrees of freedom that describe the spectrum; the search for so-called “missing
resonances” focuses on whether the spectrum can be well described by a quark model, or whether an effective theory with fewer degrees of freedom, such as a quark-diquark picture, provides a more faithful description of the baryon spectrum.

The procedure for constructing interpolating operators for baryons was described in two papers, the first of which employed a Clebsch-Gordan approach mirroring that of the continuum [46], and the second an automated method allowing a very general basis of operators to be constructed [47]. There are three double-valued irreducible representations of the cubic group, denoted \( G_{1u/g}(2) \), \( H_{u/g}(4) \) and \( G_{2u/g}(2) \), where \( g \) and \( u \) refer to positive and negative parity, respectively, and the brackets contain the dimension of the irrep. \( G_1 \) contains continuum spins \( 1/2, 3/2, \ldots \), \( H_g \) spins \( 3/2, 5/2, \ldots \) and \( G_2 \) spins \( 5/2, 7/2, \ldots \). Thus, at any fixed lattice spacing \( a \), a state corresponding to spin \( 5/2 \) has four degrees of freedom in \( H_g \) and two in \( G_2 \), with degeneracies between the energies in the two irreps. emerging in the continuum limit.

Our ability to extract the baryon resonances using the variational method was first shown in a calculation in the quenched approximation to QCD, using an anisotropic clover fermion action. This method has been extended to a calculation with 2 mass-degenerate flavors of light quarks, with gauge configurations generated on an anisotropic lattice using the Wilson fermion action. Results were obtained for two pion masses, 416(36) MeV and 578(20) MeV [48]. The lowest four energies were reported in each of the six irreducible representations of the octahedral group, at each pion mass, as illustrated in Figure 14. Most notably, evidence was found for a \( 5/2^- \) state in the pattern of negative-parity excited states, agreeing with the pattern seen in experiment, and the first time a spin-\( 5/2 \) state has been realized in a lattice QCD calculation. The “distillation” method has now been used to extend this to a calculation on the anisotropic clover lattices, for each of the nucleon, \( \Delta \) and \( \Omega \) channels; as in the case of the mesons, there is little evidence for the multi-hadron states that must be present in the spectrum.

A major goal of both the 6 GeV and future 12 GeV programs is the extraction of electromagnetic transition form factors between nucleon resonances. Once the spectrum of energies has been established, these properties can be computed in lattice QCD. Figure 15 shows the \( N - P_{11} \) transition form factors measured by CLAS, together with a lattice calculation by the Jefferson Lab group [49]. Whilst the calculation is in the quenched approximation to QCD, and at too large a pion mass to provide a reliable comparison, it demonstrates the ability of lattice QCD to go beyond the calculation of the spectrum.

5. Hadronic interactions from lattice QCD

A long-standing quest of nuclear physics has been gleaning an \textit{ab initio} understanding of how the nuclear force arises from QCD, and to rigorously compute the properties and interactions of nuclei. With the presently available computational resources, this would seem an intractable problem: the scale of the typical nuclear binding energies of a few MeV is far from the typical scale of QCD of the order of 1 GeV. There had been various attempts to gain an understanding of the nuclear force from simpler systems, beginning with an investigation of the emergence of \( \pi \) and \( \rho \) exchange in the \( B^{(*)}B^{(*)} \) system [50, 51]. However, the foundation of recent successes at studying the nature of the hadron-hadron interaction was the realization that lattice computations in the foreseeable future could be used to derive rigorous results for nuclear physics, opening up a new avenue for lattice computations that are at the core of this proposal [52].

The scattering lengths between mesons provides an important avenue in which to demonstrate our ability to understand the origin of nuclear forces, since the signal-to-noise ratio is subject to far less degradation with decreasing pion mass than baryon systems. Meson-meson scattering lengths were calculated with domain-wall fermions on \( N_f = 2 + 1 \) dynamical MILC sea configurations with pion masses down to \( m_\pi \sim 290 \text{ MeV} \) [53, 54], obtaining the results shown in Figure 16. The first prediction of the \( K\pi \) scattering lengths in both isospin channels was made possible by combining the lattice QCD calculation in the \( I = 3/2 \) channel with chiral
Figure 14. The left- and right-hand panels show the spectrum of $I = 1/2$ baryon resonance, indicated by the solid boxes, obtained on $N_f = 2$ Wilson fermion lattices at $m_\pi = 416$ and $578$ MeV respectively [48]; the errors are indicated by the vertical width of the box. The open boxes show the expected thresholds for multi-particle states.

Figure 15. Proton-Roper form factors $F_{1,2}^*$ obtained from CLAS experiments and PDG (circles) and the lattice calculation (squares, diamonds).

perturbation theory, the result of which is also shown in Figure 16. An important goal is to establish the presence of a three-hadron interaction; recently, the formalism employed above was extended to include many pions in a finite volume [56, 57]. An important outcome of this work was indeed the identification of a three-body $\pi^+\pi^+\pi^+$ interaction.

The investigation of the interaction between baryons is considerably more computationally
demanding. The signal-to-noise ratio is inherently worse for even a single baryon than is typical for a meson, and this situation is only compounded for multi-hadron states, thus requiring very high statistics. Figure 17 shows allowed regions for the scattering length in the $^1S_0$ channel as a function of the pion mass, in a hybrid DWF/Asqtad calculation analogous to that used for our studies of hadron structure [58]; the pion masses used in this study are insufficient to constrain the scattering length at the physical quark masses, but demonstrate the future promise of the method. The need for very high statistical accuracy encouraged the NPLQCD Collaboration to use the anisotropic clover lattices to measure, for the first time, the binding energy of a three-baryon $\Xi^0\Xi^0n$ system [65]: most strikingly, a window of timeslices was identified where the required statistics were lower than might naively be expected.

6. Conclusions
The last ten years have seen lattice studies incorporated as essential elements in the physics program of the laboratory, spanning the areas of hadron structure, spectroscopy and the physics of nuclei. The emergence of ever more powerful computational resources, reaching perhaps the exascale regime in the 12 GeV era, can only strengthen the role of lattice QCD in describing the
physics of Jefferson Lab, and providing the \textit{ab initio} calculations of a quality that can confront the experimental program.

For our understanding of hadron structure, the amalgam of experimental studies of transversity and generalized parton distributions, together with lattice calculations, will provide far greater insight into the distribution of mass, charge, spin and currents within a hadron than either experiment or calculation can provide alone. An important challenge will be the development of improved methods to incorporate the contributions both of the sea quarks and of the gluons to hadron structure. In the study of spectroscopy, lattice studies of the radiative transitions between light mesons will provide vital input for the GlueX experiment, and calculation of the spectrum and properties of meson and baryon resonances confronting experiment will enable us to truly understand QCD. Here an important challenge will be the developing the tools to investigate unstable resonances, and the extension of the method for understanding elastic decays to the inelastic regime. Finally, the combination of lattice calculation and effective field theory will enable the origin of the nucleon force, and of the light nuclei, to be understood.

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